Formalizing correct evidential reasoning with arguments, scenarios and probabilities

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Abstract. Artificial intelligence research on reasoning with criminal evidence in terms of arguments, hypothetical scenarios, and probabilities inspired the approach in this paper. That research showed that Bayesian Networks can be used for modeling arguments and structured hypotheses. Also well-known issues with Bayesian Network were encountered: More numbers are needed than are available, and there is a risk of misinterpretation of the graph underlying the Bayesian Network, for instance as a causal model. The formalism presented here is shown to correspond to a probabilistic interpretation, while answering these issues. The formalism is applied to key concepts in argumentative, scenario and probabilistic analyses of evidential reasoning, and is illustrated with a crime investigation example.

1 Introduction

Establishing what has happened in a crime is often not a simple task. In the literature on correct evidential reasoning, three structured analytic tools are distinguished: arguments, scenarios and probabilities [1, 8, 11]. These tools are aimed at helping organize and structure the task of evidential reasoning, thereby supporting that good conclusions are arrived at, and foreseeable mistakes are prevented.

In an *argumentative analysis*, a structured constellation of evidence, reasons and hypotheses is considered. Typically the evidence gives rise to reasons for and against the possible conclusions considered. An argumentative analysis helps the handling of such conflicts. The early twentieth century evidence scholar John Henry Wigmore is a pioneer of argumentative analyses; cf. his famous evidence charts [38, 39].

In a *scenario analysis*, different hypothetical scenarios about what has happened are considered side by side, and considered in light of the evidence. A scenario analysis helps the coherent interpretation of all evidence. Scenario analyses were the basis of legal psychology research about correct reasoning with evidence [2, 16, 37].

In a *probabilistic analysis*, it is made explicit how the probabilities of the evidence and events are related. A probabilistic analysis emphasises the various degrees of uncertainty encountered in evidential reasoning, ranging from very uncertain to very certain. Probabilistic analyses of criminal evidence go back to early forensic science in the late nineteenth century [23] and have become prominent by the statistics related to DNA profiling.

In a Netherlands-based research project,[2] artificial intelligence techniques have been used to study connections between these three tools [34]. This has resulted in the following outcomes:

- A method to manually design a Bayesian Network incorporating hypothetical scenarios and the available evidence [35];
- A case study testing the design method [35];
- A method to generate a structured explanatory text of a Bayesian Network modeled according to this method [36];
- An algorithm to extract argumentative information from a Bayesian Network modeling hypotheses and evidence [25];
- A method to incorporate argument schemes in a Bayesian Network [24].

Building on earlier work in this direction [9, 10], these results show that Bayesian Networks can be used to model arguments and structured hypotheses. Also two well-known issues encountered when using Bayesian Networks come to light:

- A Bayesian Network model typically requires many more numbers than are reasonably available;
- The graph model underlying a Bayesian Network is formally well-defined, but there is the risk of misinterpretation, for instance unwarranted causal interpretation [7] (see also [15]).

Building on the insights of the project, research has started on addressing these issues by developing an argumentation theory that connects critical arguments, coherent hypotheses and degrees of uncertainty [31, 32, 34]. The present paper expands on this work by proposing a discussion of key concepts used in argumentative, scenario and probabilistic analyses of reasoning with evidence in terms of the proposed formalism. The idea underlying this theoretical contribution is informally explained in the next section. The crime story of Alfred Hitchcock's famous film 'To Catch A Thief', featuring Cary Grant and Grace Kelly (1955) is used as an illustration.

2 General idea

The argumentation theory developed in this paper considers arguments that can be presumptive (also called ampliative), in the sense of logically going beyond their premises. Against the background of classical logic, an argument from premises P to conclusions Q goes beyond its premises when Q is not logically implied by P. Many arguments used in practice are presumptive. For instance, the prosecution may argue that a suspect was at the crime scene on the basis of a witness testimony. The fact that the witness has testified as such does not logically imply the fact that the suspect was at the crime scene. In particular, when the witness testimony is intentionally false, based on inaccurate observations or inaccurately remembered, the suspect may not have been at the crime scene at all. Denoting the witness testimony by P and the suspect being at the crime scene as Q, the

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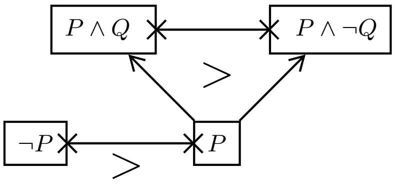

Figure 1. Some arguments

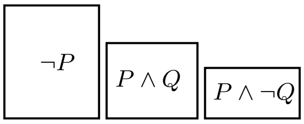

Figure 2. Some cases

argument from P to Q is presumptive since P does not logically imply Q. For presumptive arguments, it is helpful to consider the *case made by the argument*, defined as the conjunction of the premises and conclusions of the argument [29, 30]. The case made by the argument from P to Q is $P \wedge Q$, using the conjunction of classical logic. An example of a non-presumptive argument goes from $P \wedge Q$ to Q. Here Q is logically implied by $P \wedge Q$. Presumptive arguments are often defeasible [17, 26], in the sense that extending the premises may lead to the retraction of conclusions.

Figure 1 shows two presumptive arguments from the same premises P: one supports the case $P \wedge Q$, the other the case $P \wedge \neg Q$. The $>$-sign indicates that one argument makes a stronger case than the other, resolving the conflict: the argument for the case $P \wedge Q$ is stronger than that for $P \wedge \neg Q$. The figure also shows two assumptions P and $\neg P$, treated as arguments from logically tautologous premises. Here the assumption $\neg P$ makes the strongest case when compared to the assumption P. Logically such assumptions can be treated as arguments from logical truth $\top$. These four arguments—two arguments implicitly from $\top$, and two from P— make three cases: $\neg P$, $P \wedge Q$ and $P \wedge \neg Q$ (the boxes in Figure 2). The sizes of the boxes suggest a preference relation.

The comparison of arguments and of cases are closely related in our approach, which can be illustrated as follows. The idea is that a case is preferred to another case if there is an argument with premises that supports the former case more strongly than the latter case. Hence, in the example in the figures, $\neg P$ is preferred to both $P \wedge Q$ and $P \wedge \neg Q$, and $P \wedge Q$ is preferred to $P \wedge \neg Q$. Conversely, given the cases and their preferences, we can compare arguments. The argument from P to Q is stronger than from P to Q' when the best case that can be made from $P \wedge Q$ is preferred to the best case that can be made from $P \wedge Q'$.

3 Formalism and properties

We use a classical logical language L with BNF specification $\varphi ::= \top \mid \bot \mid \neg\varphi \mid \varphi \wedge \psi \mid \varphi \vee \psi \mid \varphi \leftrightarrow \psi$, and the associated classical, deductive, monotonic consequence relation, denoted $\models$. We assume a language generated by a finite set of propositional constants.

First we define case models, formalizing the idea of cases and their preferences. The cases in a case model must be logically consistent, mutually incompatible and different; and the comparison relation must be total and transitive (hence is what is called a total preorder, commonly modeling preference relations [21]).

Definition 1 *A* case model *is a pair $(C, \geq)$ with finite $C \subseteq L$, such that the following hold, for all φ, ψ and $\chi \in C$:*

1. $\not\models \neg\varphi$*;*
2. *If* $\not\models \varphi \leftrightarrow \psi$*, then* $\models \neg(\varphi \wedge \psi)$*;*
3. *If* $\models \varphi \leftrightarrow \psi$*, then* $\varphi = \psi$*;*
4. $\varphi \geq \psi$ *or* $\psi \geq \varphi$ *;*
5. *If* $\varphi \geq \psi$ *and* $\psi \geq \chi$*, then* $\varphi \geq \chi$*.*

The strict weak order $>$ standardly associated with a total preorder $\geq$ is defined as $\varphi > \psi$ if and only if it is not the case that $\psi \geq \varphi$ (for φ and $\psi \in C$). When $\varphi > \psi$, we say that φ is (strictly) preferred to ψ. The associated equivalence relation $\sim$ is defined as $\varphi \sim \psi$ if and only if $\varphi \geq \psi$ and $\psi \geq \varphi$.

Example. Figure 2 shows a case model with cases $\neg P$, $P \wedge Q$ and $P \wedge \neg Q$. $\neg P$ is (strictly) preferred to $P \wedge Q$, which in turn is preferred to $P \wedge \neg Q$.

Although the preference relations of case models are qualitative, they correspond to the relations that can be represented by real-valued functions.

Corollary 2 *Let $C \subseteq L$ be finite with elements that are logically consistent, mutually incompatible and different (properties 1, 2 and 3 in the definition of case models). Then the following are equivalent:*

1. $(C, \geq)$ *is a case model;*
2. $\geq$ *is numerically representable, i.e., there is a real valued function v on C such that for all φ and $\psi \in C$, $\varphi \geq \psi$ if and only if $v(\varphi) \geq v(\psi)$.*

The function v can be chosen with only positive values, or even with only positive integer values.

Proof. It is a standard result in order theory that total preorders on finite (or countable) sets are the ones that are representable by a real-valued function [21]. QED

Corollary 3 *Let $C \subseteq L$ be finite with elements that are logically consistent, mutually incompatible and different (properties 1, 2 and 3 in the definition of case models). Then the following are equivalent:*

1. $(C, \geq)$ *is a case model;*
2. $\geq$ *is numerically representable by a probability function p on the algebra generated by C such that for all φ and $\psi \in C$, $\varphi \geq \psi$ if and only if $p(\varphi) \geq p(\psi)$.*

Proof. Pick a representing real-valued function v with only positive values as in the previous corollary, and (for elements of C) define the values of p as those of v divided by the sum of the v-values of all cases; then extend to the algebra generated by C. QED

Next we define arguments. Arguments are from premises $\varphi \in L$ to conclusions $\psi \in L$.

Definition 4 *An* argument *is a pair (φ, ψ) with φ and $\psi \in L$. The sentence φ expresses the argument's premises, the sentence ψ its conclusions, and the sentence $\varphi \wedge \psi$ the* case made *by the argument. Generalizing, a sentence $\chi \in L$ is a* premise *of the argument when $\varphi \models \chi$, a* conclusion *when $\psi \models \chi$, and a* position *in the case made by the argument when $\varphi \wedge \psi \models \chi$. An argument (φ, ψ) is* (properly) presumptive *when $\varphi \not\models \psi$; otherwise* non-presumptive*. An argument (φ, ψ) is an* assumption *when $\models \varphi$, i.e., when its premises are logically tautologous.*

Note our use of the plural for an argument's premises, conclusions and positions. This terminological convention allows us to speak of the premises $\mathtt{p}$ and $\neg\mathtt{q}$ and conclusions $\mathtt{r}$ and $\neg\mathtt{s}$ of the argument $(\mathtt{p} \wedge \neg\mathtt{q}, \mathtt{r} \wedge \neg\mathtt{s})$. Also the convention fits our non-syntactic definitions, where for instance an argument with premise χ also has logically equivalent sentences such as $\neg\neg\chi$ as a premise.

Coherent arguments are defined as arguments that make a case logically implied by a case in the case model.

Definition 5 *Let $(C, \geq)$ be a case model. Then we define, for all φ and $\psi \in L$:*

$(C, \geq) \models (\varphi, \psi)$ *if and only if* $\exists \omega \in C$: $\omega \models \varphi \wedge \psi$.

We then say that the argument from φ to ψ is coherent *with respect to the case model. We say that a coherent argument from φ to ψ is* conclusive *when all cases implying the premises also imply the conclusions.*

Example (continued). In the case model of Figure 2, the arguments from $\top$ to $\neg P$ and to P, and from P to Q and to $\neg Q$ are coherent and not conclusive in the sense of this definition. Denoting the case model as $(C, \geq)$, we have $(C, \geq) \models (\top, \neg P)$, $(C, \geq) \models (\top, P)$, $(C, \geq) \models (P, Q)$ and $(C, \geq) \models (P, \neg Q)$. The arguments from a case (in the case model) to itself, such as from $\neg P$ to $\neg P$, or from $P \wedge Q$ to $P \wedge Q$ are conclusive. The argument $(P \vee R, P)$ is also conclusive in this case model, since all $P \vee R$-cases are P-cases. Similarly, $(P \vee R, P \vee S)$ is conclusive.

The notion of presumptive validity considered here is based on the idea that some arguments make a better case than other arguments from the same premises. More precisely, an argument is presumptively valid if there is a case in the case model implying the case made by the argument that is at least as preferred as all cases implying the premises.

Definition 6 *Let $(C, \geq)$ be a case model. Then we define, for all φ and $\psi \in L$:*

$(C, \geq) \models \varphi \rightsquigarrow \psi$ *if and only if* $\exists \omega \in C$:

1. $\omega \models \varphi \wedge \psi$*; and*
2. $\forall \omega' \in C$: *if* $\omega' \models \varphi$*, then* $\omega \geq \omega'$.

We then say that the argument from φ to ψ is (presumptively) valid *with respect to the case model. A presumptively valid argument is* (properly) defeasible*, when it is not conclusive.*

Example (continued). In the case model of Figure 2, the arguments from $\top$ to $\neg P$, and from P to Q are presumptively valid in the sense of this definition. Denoting the case model as $(C, \geq)$, we have formally that $(C, \geq) \models \top \rightsquigarrow \neg P$ and $(C, \geq) \models P \rightsquigarrow Q$. The coherent arguments from $\top$ to P and from P to $\neg Q$ are not presumptively valid in this sense.

Corollary 7 *1. Conclusive arguments are coherent, but there are case models with a coherent, yet inconclusive argument;*

2. Conclusive arguments are presumptively valid, but there are case models with a presumptively valid, yet inconclusive argument;

3. Presumptively valid arguments are coherent, but there are case models with a coherent, yet presumptively invalid argument.

The next proposition provides key logical properties of this notion of presumptive validity. Many have been studied for nonmonotonic inference relations [13, 14, 27]. Given a case model $(C, \geq)$, we write $\varphi \mid\!\sim \psi$ for $(C, \geq) \models \varphi \rightsquigarrow \psi$. We write $C(\varphi)$ for the set $\{\omega \in C \mid \omega \models \varphi\}$.

(LE), for Logical Equivalence, expresses that in a valid argument the premises and the conclusions can be replaced by a classical equivalent (in the sense of $\models$). (Cons), for Consistency, expresses that the conclusions of presumptively valid arguments must be consistent. (Ant), for Antededence, expresses that when certain premises validly imply a conclusion, the case made by the argument is also validly implied by these premises. (RW), for Right Weakening, expresses that when the premises validly imply a composite conclusion also the intermediate conclusions are validly implied. (CCM), for Conjunctive Cautious Monotony, expresses that the case made by a valid argument is still validly implied when an intermediate conclusion is added to the argument's premises. (CCT), for Conjunctive Cumulative Transitivity, is a variation of the related property Cumulative Transitivity property (CT, also known as Cut). (CT)—extensively studied in the literature—has $\varphi \mid\!\sim \chi$ instead of $\varphi \mid\!\sim \psi \wedge \chi$ as a consequent. The variation is essential in our setting where the (And) property is absent (If $\varphi \mid\!\sim \psi$ and $\varphi \mid\!\sim \chi$, then $\varphi \mid\!\sim \psi \wedge \chi$). Assuming (Ant), (CCT) expresses the validity of chaining valid implication from φ via the case made in the first step $\varphi \wedge \psi$ to the case made in the second step $\varphi \wedge \psi \wedge \chi$. (See [29, 30], introducing (CCT).)

Proposition 8 *Let $(C, \geq)$ be a case model. For all φ, ψ and $\chi \in L$:*

(LE)	*If* $\varphi \mid\!\sim \psi$, $\models \varphi \leftrightarrow \varphi'$ *and* $\models \psi \leftrightarrow \psi'$*, then* $\varphi' \mid\!\sim \psi'$.
(Cons)	$\varphi \not\mid\!\sim \bot$.
(Ant)	*If* $\varphi \mid\!\sim \psi$*, then* $\varphi \mid\!\sim \varphi \wedge \psi$.
(RW)	*If* $\varphi \mid\!\sim \psi \wedge \chi$*, then* $\varphi \mid\!\sim \psi$.
(CCM)	*If* $\varphi \mid\!\sim \psi \wedge \chi$*, then* $\varphi \wedge \psi \mid\!\sim \chi$.
(CCT)	*If* $\varphi \mid\!\sim \psi$ *and* $\varphi \wedge \psi \mid\!\sim \chi$*, then* $\varphi \mid\!\sim \psi \wedge \chi$.

Proof. (LE): Direct from the definition. (Cons): Otherwise there would be an inconsistent element of C, contradicting the definition of a case model. (Ant): When $\varphi \mid\!\sim \psi$, there is an ω with $\omega \models \varphi \wedge \psi$ that is $\geq$-maximal in $C(\varphi)$. Then also $\omega \models \varphi \wedge \varphi \wedge \psi$, hence $\varphi \mid\!\sim \varphi \wedge \psi$. (RW): When $\varphi \mid\!\sim \psi \wedge \chi$, there is an $\omega \in C$ with $\omega \models \varphi \wedge \psi \wedge \chi$ that is maximal in $C(\varphi)$. Since then also $\omega \models \varphi \wedge \psi$, we find $\varphi \mid\!\sim \psi$. (CCM): By the assumption, we have an $\omega \in C$ with $\omega \models \varphi \wedge \psi \wedge \chi$ that is maximal in $C(\varphi)$. Since $C(\varphi \wedge \psi) \subseteq C(\varphi)$, ω is also maximal in $C(\varphi \wedge \psi)$, and we find $\varphi \wedge \psi \mid\!\sim \chi$. (CCT): Assuming $\varphi \mid\!\sim \psi$, there is an $\omega \in C$ with $\omega \models \varphi \wedge \psi$, maximal in $C(\varphi)$. Assuming also $\varphi \wedge \psi \mid\!\sim \chi$, there is an $\omega' \in C$ with $\omega \models \varphi \wedge \psi \wedge \chi$, maximal in $C(\varphi \wedge \psi)$. Since $\omega \in C(\varphi \wedge \psi)$, we find $\omega' \geq \omega$. By transitivity of $\geq$, and the maximality of ω in $C(\varphi)$, we therefore have that ω' is maximal in $C(\varphi)$. As a result, $\varphi \mid\!\sim \psi \wedge \chi$. QED

We speak of coherent premises when the argument from the premises to themselves is coherent. The following proposition provides some equivalent characterizations of coherent premises.

Proposition 9 *Let $(C, \geq)$ be a case model. The following are equivalent, for all $\varphi \in L$:*

1. $\varphi \mid\!\sim \varphi$*;*
2. $\exists \omega \in C : \omega \models \varphi$ *and* $\forall \omega' \in C$*: If* $\omega' \models \varphi$*, then* $\omega \geq \omega'$*;*
3. $\exists \omega \in C : \varphi \mid\!\sim \omega$.
4. $\exists \omega \in C : \omega \models \varphi$.

Proof. 1 and 2 are equivalent by the definition of $\mid\!\sim$. Assume 2. Then there is a $\geq$-maximal element ω of $C(\varphi)$. By the definition of $\mid\!\sim$, then $\varphi \mathrel{|\!\sim} \omega$; proving 3. Assume 3. Then there is a $\geq$-maximal element ω' of $C(\varphi)$ with $\omega' \models \varphi \wedge \omega$. For this ω' also $\omega' \models \varphi$, showing 2. 4 logically follows from 2. 4 implies 2 since L is a language that generated by finitely many propositional constants. QED

Corollary 10 *Let $(C, \geq)$ be a case model. Then all coherent arguments have coherent premises and all presumptively valid arguments have coherent premises.*

We saw that, in the present approach, premises are coherent when they are logically implied by a case in the case model. As a result, generalisations of coherent premises are again coherent; cf. the following corollary.

Corollary 11 *Let $(C, \geq)$ be a case model. Then:*

If $\varphi \mathrel{|\!\sim} \varphi$ and $\varphi \models \psi$, then $\psi \mathrel{|\!\sim} \psi$.

We now consider some properties that use a subset L^* of the language L. The set L^* consists of the logical combinations of the cases of the case model using negation, conjunction and logical equivalence (cf. the algebra underlying probability functions [21]). L^* is the set of *case expressions* associated with a case model.

(Coh), for Coherence, expresses that coherent premises correspond to a consistent case expression implying the premises. (Ch), for Choice, expresses that, given two coherent case expressions, at least one of three options follows validly: the conjunction of the case expression, or the conjunction of one of them with the negation of the other. (OC), for Ordered Choice, expresses that preferred choices between case expressions are transitive. Here we say that a case expression is a *preferred choice* over another, when the former follows validly from the disjunction of both.

Definition 12 *Let $(C, \geq)$ be a case model, $\varphi \in L$, and $\omega \in C$. Then ω expresses a* preferred case *of φ if and only if $\varphi \mathrel{|\!\sim} \omega$.*

Proposition 13 *Let $(C, \geq)$ be a case model, and $L^* \subseteq L$ the closure of C under negation, conjunction and logical equivalence. Writing $\mathrel{|\!\sim}^*$ for the restriction of $\mathrel{|\!\sim}$ to L^*, we have, for all φ, ψ and $\chi \in L^*$:*

(Coh) $\varphi \mathrel{|\!\sim} \varphi$ *if and only if* $\exists \varphi^* \in L^*$ *with* $\varphi^* \not\models \bot$ *and* $\varphi^* \models \varphi$;
(Ch) *If* $\varphi \mathrel{|\!\sim}^* \varphi$ *and* $\psi \mathrel{|\!\sim}^* \psi$, *then* $\varphi \vee \psi \mathrel{|\!\sim}^* \neg\varphi \wedge \psi$ *or* $\varphi \vee \psi \mathrel{|\!\sim}^* \varphi \wedge \psi$ *or* $\varphi \vee \psi \mathrel{|\!\sim}^* \varphi \wedge \neg\psi$;
(OC) *If* $\varphi \vee \psi \mathrel{|\!\sim}^* \varphi$ *and* $\psi \vee \chi \mathrel{|\!\sim}^* \psi$, *then* $\varphi \vee \chi \mathrel{|\!\sim}^* \varphi$.

Proof. (Coh): By Proposition 9, $\varphi \mathrel{|\!\sim} \varphi$ if and only if there is an $\omega \in C$ with $\omega \models \varphi$. The property (Coh) follows since $C \subseteq L^*$ and, for all consistent $\varphi^* \in L^*$, there is an $\omega \in C$ with $\omega \models \varphi^*$.
(Ch): Consider sentences φ and $\psi \in L^*$ with $\varphi \mathrel{|\!\sim}^* \varphi$ and $\psi \mathrel{|\!\sim}^* \psi$. Then, by Corollary 11, $\varphi \vee \psi \mathrel{|\!\sim} \varphi \vee \psi$. By Proposition 9, there is an $\omega \in C$, with $\omega \models \varphi \vee \psi$. The sentences φ and ψ are elements of L^*, hence also the sentences $\varphi \wedge \neg\psi$, $\varphi \wedge \psi$ and $\neg\varphi \wedge \psi \in L^*$. All are logically equivalent to disjunctions of elements of C (possibly the empty disjunction, logically equivalent to $\bot$). Since $\omega \models \varphi \vee \psi$, $\models \varphi \vee \psi \leftrightarrow (\varphi \wedge \neg\psi) \vee (\varphi \wedge \psi) \vee (\neg\varphi \wedge \psi)$, and the elements of C are mutually incompatible, we have $\omega \models \varphi \wedge \neg\psi$ or $\omega \models \varphi \wedge \psi$ or $\omega \models \neg\varphi \wedge \psi$. By Proposition 9, it follows that $\varphi \vee \psi \mathrel{|\!\sim}^* \neg\varphi \wedge \psi$ or $\varphi \vee \psi \mathrel{|\!\sim}^* \varphi \wedge \psi$ or $\varphi \vee \psi \mathrel{|\!\sim}^* \varphi \wedge \neg\psi$.
(OC): By $\varphi \vee \psi \mathrel{|\!\sim}^* \varphi$, there is an $\omega \models \varphi$ maximal in $C(\varphi \vee \psi)$. By $\psi \vee \chi \mathrel{|\!\sim}^* \psi$, there is an $\omega' \models \psi$ maximal in $C(\psi \vee \chi)$. Since $\omega \models \varphi$, $\omega \in C(\varphi \vee \chi)$. Since $\omega' \models \psi$, $\omega' \in C(\varphi \vee \psi)$, hence $\omega \geq \omega'$. Hence ω is maximal in $C(\varphi \vee \chi)$, hence $\varphi \vee \chi \mathrel{|\!\sim} \varphi$. Since $\chi \in L^*$, $\varphi \vee \chi \mathrel{|\!\sim}^* \varphi$. QED

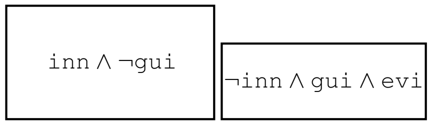

Figure 3. A case model for presumption

4 A formal analysis of some key concepts

We now use the formalism of case models and presumptive validity above for a discussion of some key concepts associated with the argumentative, scenario and probabilistic analysis of evidential reasoning.

4.1 Arguments

In an argumentative analysis, it is natural to classify arguments with respect to the nature of the support their premises give their conclusions. We already defined non-presumptive and (properly) presumptive arguments (Definition 4), and—with respect to a case model—presumptively valid and (properly) defeasible arguments (Definition 6). We illustrate these notions in an example about the presumption of innocence.

Let $\mathtt{inn}$ denote that a suspect is innocent, and $\mathtt{gui}$ that he is guilty. Then the argument $(\mathtt{inn}, \neg\mathtt{gui})$ is (properly) presumptive, since $\mathtt{inn} \not\models \neg\mathtt{gui}$. The argument $(\mathtt{inn} \wedge \neg\mathtt{gui}, \neg\mathtt{gui})$ is non-presumptive, since $\mathtt{inn} \wedge \neg\mathtt{gui} \models \neg\mathtt{gui}$.

Presumptive validity and (proper) defeasibility are illustrated using a case model. Consider the case model with two cases $\mathtt{inn} \wedge \neg\mathtt{gui}$ and $\neg\mathtt{inn} \wedge \mathtt{gui} \wedge \mathtt{evi}$ with the first case preferred to the second (Figure 3; the surface of the cases measures their preference). Here $\mathtt{evi}$ denotes evidence for the suspect's guilt. Then the (properly) presumptive argument $(\mathtt{inn}, \neg\mathtt{gui})$ is presumptively valid with respect to this case model since the conclusion $\neg\mathtt{gui}$ follows in the case $\mathtt{inn} \wedge \neg\mathtt{gui}$ that is a preferred case of the premise $\mathtt{inn}$. The argument is conclusive since there are no other cases implying $\mathtt{inn}$. The argument $(\top, \mathtt{inn})$—in fact an assumption now that its premises are tautologous—is presumptively valid since $\mathtt{inn}$ follows in the preferred case $\mathtt{inn} \wedge \neg\mathtt{gui}$. This shows that the example represents what is called the presumption of innocence, when there is no evidence. This argument is (properly) defeasible since in the other case of the argument's premises the conclusion does not follow. In fact, the argument $(\mathtt{evi}, \mathtt{inn})$ is not coherent since there is no case in which both $\mathtt{evi}$ and $\mathtt{inn}$ follow. The argument $(\mathtt{evi}, \mathtt{gui})$ is presumptively valid, even conclusive.

In argumentative analyses, different kinds of argument attack are considered. John Pollock made the famous distinction between two kinds of—what he called—argument defeaters [17, 18]. A rebutting defeater is a reason for a conclusion that is the opposite of the conclusion of the attacked argument, whereas an undercutting defeater is a reason that attacks not the conclusion itself, but the connection between reason and conclusion. Joseph Raz made a related famous distinction of exclusionary reasons that always prevail, independent of the strength of competing reasons [19] (see also [20]).

We propose the following terminology.

Definition 14 *Let $(C, \geq)$ be a case model, and (φ, ψ) a presumptively valid argument. Then circumstances χ are* undercutting *when $(\varphi \wedge \chi, \psi)$ is not presumptively valid. Undercutting circumstances are* rebutting *when $(\varphi \wedge \chi, \neg\psi)$ is presumptively valid; otherwise they are* properly undercutting. *Undercutting circumstances are* excluding *when $(\varphi \wedge \chi, \psi)$ is not coherent.*

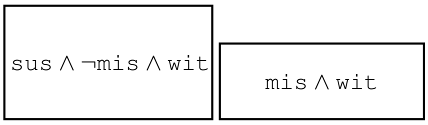

Figure 4. A case model for proper undercutting

Continuing the example of the case model illustrated in Figure 3, we find the following. The circumstances $\mathtt{evi}$ undercut the presumptively valid argument $(\top, \mathtt{inn})$ since $(\mathtt{evi}, \mathtt{inn})$ is not presumptively valid. In fact, these circumstances are excluding since $(\mathtt{evi}, \mathtt{inn})$ is not coherent. The circumstances are also rebutting since the argument for the opposite conclusion $(\mathtt{evi}, \neg\mathtt{inn})$ is presumptively valid.

Proper undercutting can be illustrated with an example about a lying witness. Consider a case model with these two cases:

1: $\mathtt{sus} \wedge \neg\mathtt{mis} \wedge \mathtt{wit}$
2: $\mathtt{mis} \wedge \mathtt{wit}$

In the cases, there is a witness testimony ($\mathtt{wit}$) that the suspect was at the crime scene ($\mathtt{sus}$). In Case 1, the witness was not misguided ($\neg\mathtt{mis}$), in Case 2 he was. In Case 1, the suspect was indeed at the crime scene; in Case 2, the witness was misguided and it is unspecified whether the suspect was at the crime scene or not. In the case model, Case 1 is preferred to Case 2 (Figure 4), representing that witnesses are usually not misguided.

Since Case 1 is a preferred case of $\mathtt{wit}$, the argument $(\mathtt{wit}, \mathtt{sus})$ is presumptively valid: the witness testimony provides a presumptively valid argument for the suspect having been at the crime scene. The argument's conclusion can be strengthened to include that the witness was not misguided. Formally, this is expressed by saying that $(\mathtt{wit}, \mathtt{sus} \wedge \neg\mathtt{mis})$ is a presumptively valid argument. There are circumstances undercutting the argument $(\mathtt{wit}, \mathtt{sus})$, namely when the witness was misguided after all ($\mathtt{mis}$). This can be seen by considering that Case 2 is the only case in which $\mathtt{wit} \wedge \mathtt{mis}$ follows, hence is preferred. Since $\mathtt{sus}$ does not follow in Case 2, the argument $(\mathtt{wit} \wedge \mathtt{mis}, \mathtt{sus})$ is not presumptively valid. The misguidedness is not rebutting, hence properly undercutting since $(\mathtt{wit} \wedge \mathtt{mis}, \neg\mathtt{sus})$ is not presumptively valid. The misguidedness is excluding since the argument $(\mathtt{wit} \wedge \mathtt{mis}, \mathtt{sus})$ is not even coherent.

Arguments can typically be chained, namely when the conclusion of one is a premise of another. For instance when there is evidence ($\mathtt{evi}$) that a suspect is guilty of a crime ($\mathtt{gui}$), the suspect's guilt can be the basis of punishing the suspect ($\mathtt{pun}$). For both steps there are typical undercutting circumstances. The step from the evidence to guilt is blocked when there is an alibi ($\mathtt{ali}$), and the step from guilt to punishing is blocked when there are grounds of justification ($\mathtt{jus}$), such as force majeure. A case model with three cases can illustrate such chaining:

1: $\mathtt{pun} \wedge \mathtt{gui} \wedge \mathtt{evi}$
2: $\neg\mathtt{pun} \wedge \mathtt{gui} \wedge \mathtt{evi} \wedge \mathtt{jus}$
3: $\neg\mathtt{gui} \wedge \mathtt{evi} \wedge \mathtt{ali}$

In the case model, Case 1 is preferred to Case 2 and Case 3, modeling that the evidence typically leads to guilt and punishing, unless there are grounds for justification (Case 2) or there is an alibi (Case 3). Cases 2 and 3 are preferentially equivalent.

In this case model, the following arguments are presumptively valid:

1: $(\mathtt{evi}, \mathtt{gui})$
2: $(\mathtt{gui}, \mathtt{pun})$
3: $(\mathtt{evi}, \mathtt{gui} \wedge \mathtt{pun})$

Arguments 1 and 3 are presumptively valid since Case 1 is the preferred case among those in which $\mathtt{evi}$ follows; Argument 2 is since Case 1 is the preferred case among those in which $\mathtt{gui}$ follows. By chaining arguments 1 and 2, the case for $\mathtt{gui} \wedge \mathtt{pun}$ can be based on the evidence $\mathtt{evi}$ as in Argument 3.

The following arguments are not presumptively valid in this case model:

4: $(\mathtt{evi} \wedge \mathtt{ali}, \mathtt{gui})$
5: $(\mathtt{gui} \wedge \mathtt{jus}, \mathtt{pun})$

This shows that Arguments 1 and 2 are undercut by circumstances $\mathtt{ali}$ and $\mathtt{jus}$, respectively. As expected, chaining these arguments fails under both of these circumstances, as shown by the fact that these two arguments are not presumptively valid:

6: $(\mathtt{evi} \wedge \mathtt{ali}, \mathtt{gui} \wedge \mathtt{pun})$
7: $(\mathtt{evi} \wedge \mathtt{jus}, \mathtt{gui} \wedge \mathtt{pun})$

But the step to guilt can be made when there are grounds for justification. Formally, this can be seen by the presumptive validity of this argument:

8: $(\mathtt{evi} \wedge \mathtt{jus}, \mathtt{gui})$

4.2 Scenarios

In the literature on scenario analyses, several notions are used in order to analyze the 'quality' of the scenarios considered. Three notions are prominent: a scenario's consistency, a scenario's completeness and a scenario's plausibility [16, 37]. In this literature, these notions are part of an informally discussed theoretical background, having prompted recent work in AI & Law on formalizing these notions [3, 33, 36]. A scenario is consistent when it does not contain contradictions. For instance, a suspect cannot be both at home and at the crime scene. A scenario is complete when all relevant elements are in the scenario. For instance, a murder scenario requires a victim, an intention and premeditation. A scenario is plausible when it fits commonsense knowledge about the world. For instance, in a murder scenario, a victim's death caused by a shooting seems a plausible possibility. We now propose a formal treatment of these notions using the formalism presented.

The consistency of a scenario can simply be taken to correspond to logical consistency. A more interesting, stronger notion of scenario-consistency uses the world knowledge takes represented in a case model and defines a scenario as scenario-consistent when it is a logically consistent coherent assumption. Formally, writing S for the scenario, S is scenario-consistent when S is logically consistent and the argument $(\top, S)$ is coherent, i.e., there is a case in the case model logically implying S.

The completeness of a scenario can here be defined using a notion of maximally specific conclusions, as follows.

Definition 15 *Let $(C, \geq)$ be a case model, and (φ, ψ) a presumptively valid argument. Then the case made by the argument (i.e., $\varphi \wedge \psi$) is an* extension *of φ when there is no presumptively valid argument from φ that makes a case that is logically more specific.*

For instance, consider a case model in which the case vic ∧ int ∧ pre ∧ evi is a preferred case of evi. The case expresses a situation in which there is evidence (evi) for a typical murder: there is a victim (vic), there was the intention to kill (int), and there was premeditation (pre). In such a case model, this case is an extension of the evidence evi. A scenario can now be considered complete with respect to certain evidence when the scenario conjoined with the evidence is its own extension. In the example, the sentence vic ∧ int ∧ pre is a complete scenario given evi as the scenario conjoined with the evidence is its own extension. The sentence vic ∧ int is not a complete scenario given evi, as the extension of vic ∧ int ∧ evi also implies pre.

A scenario can be treated as plausible (given a case model) when it is a presumptively valid conclusion of the evidence. Continuing the example, the complete scenario vic ∧ int ∧ pre is then plausible given evi, but also subscenarios such as vic ∧ int (leaving the premeditation unspecified) and int ∧ pre (with no victim, only intention and premeditation). This notion of a scenario's plausibility depends on the evidence, in contrast with the mentioned literature [16, 37], where plausibility is treated as being independent from the evidence. The present proposal includes an evidence-independent notion of plausibility, by considering a scenario as plausible—independent of the evidence—when it is plausible given no evidence, i.e., when the scenario is a presumptively valid assumption. In the present setting, plausibility can be connected to the preference ordering on cases given the evidence, when scenarios are complete. A complete scenario is than more plausible than another given the evidence when the former is preferred to the latter.

4.3 Probabilities

The literature on the probabilistic analysis of reasoning with evidence uses the probability calculus as formal background. A key formula is the well-known Bayes' theorem, stating that for events H and E the following relation between probabilities holds:

$$\Pr(H|E) = \frac{\Pr(E|H)}{\Pr(E)} \cdot \Pr(H)$$

Thinking of H as a hypothesis and E as evidence, here the posterior probability $\Pr(H|E)$ of the hypothesis given the evidence can be computed by multiplying the prior probability $\Pr(H)$ and the Bayes factor $\Pr(E|H)/\Pr(E)$.

We saw that the preferences of our case models are exactly those that can be realized by probability functions over the cases in the model (Corollary 3). Given a realization of a case model, key concepts defined in terms of the case model translate straightforwardly to the probabilistic setting. For instance, a preferred case (given certain premises) has maximal probability (conditional on these premises) among the cases from which the premises follow. Also the premises provide a conclusive argument for a case if there is exactly one case from which the premises follow, hence if the probability of the case given the premises is equal to 1. Also, clearly, Bayes' theorem holds for any such probabilistic realization of our case models.

A formula that is especially often encountered in the literature on evidential reasoning is the following odds version of Bayes' theorem:

$$\frac{\Pr(H|E)}{\Pr(\neg H|E)} = \frac{\Pr(E|H)}{\Pr(E|\neg H)} \cdot \frac{\Pr(H)}{\Pr(\neg H)}$$

Here the posterior odds $\Pr(H|E)/\Pr(\neg H|E)$ of the hypothesis given the evidence is found by multiplying the prior odds $\Pr(H)/\Pr(\neg H)$ with the likelihood ratio $\Pr(E|H)/\Pr(E|\neg H)$. This formula is important since the likelihood ratio can sometimes be estimated, for instance in the case of DNA evidence. In fact, it is a key lesson in probabilistic approaches to evidential reasoning that the evidential value of evidence, as measured by a likelihood ratio, does not by itself determine the posterior probability of the hypothesis considered. As the formula shows, the prior probability of the hypothesis is needed to determine the posterior probability given the likelihood ratio. Just as Bayes' theorem, the likelihood ratio obtains in a probabilistic realization of a case model in our sense.

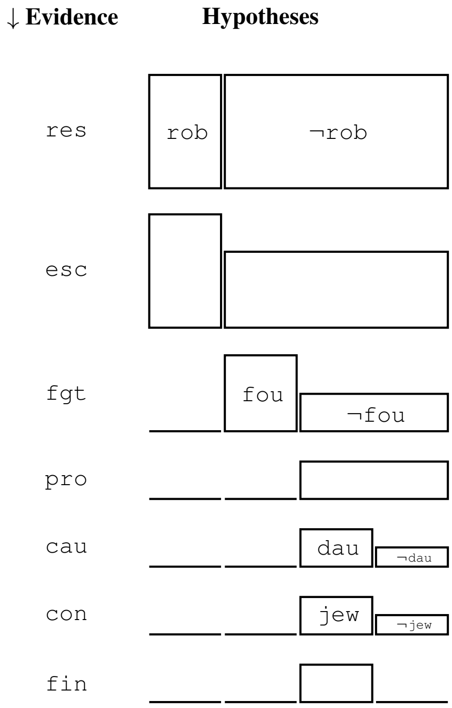

Figure 5. Example: Hitchcock's 'To Catch A Thief'

5 Example: Alfred Hitchcock's 'To Catch A Thief'

As an example of the development of evidential reasoning in which gradually information is collected, we discuss the crime investigation story that is the backbone of Alfred Hitchcock's 'To Catch A Thief', otherwise—what Hitchcock himself referred to as—a lightweight story about a French Riviera love affair, starring Grace Kelly and Cary Grant. In the film, Grant plays a former robber Robie, called 'The Cat' because of his spectacular robberies, involving the climbing of high buildings. At the beginning of the film, new 'The Cat'-like thefts have occurred. Because of this resemblance with Robie's style (the first evidence considered, denoted in what follows as res), the police consider the hypothesis that Robie is again the thief (rob), and also that he is not (¬rob). Figure 5 provides a graphical representation of the investigation. The first row shows the situation after the first evidence res, mentioned on the left side of the figure, with the two hypothical conclusions rob and ¬rob represented as rectangles. A rectangle's height suggests the strength of the argument from the accumulated evidence to the hypothesis. Here the arguments from res to rob and ¬rob are of comparable strength.

When the police confront Robie with the new thefts, he escapes with the goal to catch the real thief. By this second evidence (esc), the hypothesis rob becomes more strongly supported than its opposite ¬rob. In the figure, the second row indicates the situation after the two pieces of evidence are available. As indicated by the rectangles of different heights, the argument from the accumulated evidence res∧esc to rob is stronger than that from the same premises to ¬rob. Rectangles in a column in the figure represent corresponding hypotheses. Sentences shown in a corresponding hypothesis in a higher row are not repeated.

Robie sets a trap for the real thief, resulting in a nightly fight on the roof with Foussard who falls and dies (fgt). The police consider this strong evidence for the hypothesis that Foussard is the thief (fou), but not conclusive so also the opposite hypothesis is considered coherent (¬fou). In the figure (third row marked fgt) the hypothesis ¬rob is split into two hypotheses: one rectangle representing ¬rob∧fou, the other ¬rob∧¬fou. With the accumulated evidence res∧esc ∧fgt as premises, the hypothesis ¬rob∧fou is more strongly supported than the hypothesis ¬rob∧¬fou. The police no longer believe that Robie is the thief. This is indicated by the line on the left of the third row in the figure. The premises res∧esc ∧fgt do not provide support for the hypothesis rob; or, in the terminology of this paper: the argument from premises res∧esc ∧fgt to conclusion rob is not coherent.

Robie points out that Foussard cannot be the new incarnation of 'The Cat', as he had a prosthetic wooden leg (pro). In other words, the argument from res∧esc ∧fgt∧pro to ¬rob∧fou is not coherent. (Cf. the second line in the fourth row of the figure, corresponding to the hypothesis that Foussard is the thief.)

Later in the film, Foussard's daughter is caught in the act (cau), providing very strong support for the hypothesis that the daughter is the new cat (dau). The argument from res∧esc ∧fgt∧pro∧cau to dau is stronger than to ¬dau.

In her confession (con), Foussard's daughter explains where the jewelry stolen earlier can be found, adding some specific information to the circumstances of her crimes (jew). The argument from res∧esc ∧fgt∧pro∧cau ∧con to dau ∧ jew is stronger than to ¬dau ∧ ¬jew.

The police find the jewelry at the indicated place (fin) and there is no remaining doubt about the hypothesis that Foussard's daughter is the thief. The argument from res∧esc ∧fgt∧pro∧cau∧con ∧fin to ¬dau∧ ¬jew is incoherent, as indicated by the line on the right of the bottom row of the figure. In the only remaining hypothesis, Foussard's daughter is the thief, and not Robie, not Foussard. In other words, the argument from res∧esc ∧fgt∧pro∧cau∧con ∧jew to ¬rob∧¬fou∧dau is conclusive.

We can use the constructions of the representation theorem to develop a case model representing the arguments discussed in the example. We distinguish 7 cases, as follows:

1. rob
   ∧ res∧esc
2. ¬rob∧fou
   ∧ res∧esc∧fgt
3. ¬rob∧¬fou∧dau∧jew
   ∧ res∧esc∧fgt∧pro∧cau∧con∧fin
4. ¬rob∧¬fou∧¬dau∧¬jew
   ∧ res∧esc∧fgt∧pro∧cau∧con
5. ¬rob
   ∧res∧¬esc
6. ¬rob∧¬fou
   ∧res∧esc∧¬fgt
7. ¬rob∧¬fou∧¬dau
   ∧res∧esc∧fgt∧pro∧¬cau

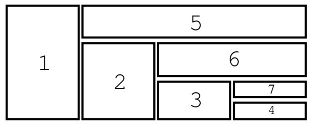

Figure 6. Case model for the example

Cases 1 to 4 are found as follows. First the properties of the four main hypotheses are accumulated (rob, ¬rob∧fou, ¬rob∧¬fou∧dau∧jew, ¬rob∧¬fou∧¬dau∧¬jew). Then these are conjoined with the maximally specific accumulated evidence that provide a coherent argument for them (res∧esc, res∧esc∧fgt, res∧esc∧fgt∧pro∧cau∧con∧fin, res∧ esc∧fgt∧pro∧cau∧con). The cases 5 to 7 complete the case model. Case 5 is the hypothetical case that Robie is not the thief, that there is resemblance, and the Robie does not escape. In case 6, Robie and Foussard are not the thieves, and there is no fight. In case 7, Robie, Foussard and his daughter are not the thieves, and she is not caught in the act. Note that the cases are consistent and mutually exclusive.

Figure 6 shows the 7 cases of the model. The sizes of the rectangles represent the preferences. The preference relation has the following equivalence classes, ordered from least preferred to most preferred:

1. Cases 4 and 7;
2. Case 3;
3. Cases 2 and 6;
4. Cases 1 and 5.

The discussion of the arguments, their coherence, conclusiveness and validity presented semi-formally above fits this case model. For instance, the argument from the evidential premises res∧esc to the hypothesis rob is presumptively valid in this case model since Case 1 is the only case implying the case made by the argument. It is not conclusive since also the argument from these same premises to ¬rob is coherent. The latter argument is not presumptively valid since all cases implying the premises have lower preference than Case 1. The argument from res∧esc∧fgt to rob is incoherent as there is no case in which the premises and the conclusion follow. Also arguments that do not start from evidential premises can be evaluated. For instance, the argument from the premise (not itself evidence) dau to jew is conclusive since in the only case implying the premises (Case 3) the conclusion follows. Finally we find the conclusive argument from premises res∧esc∧fgt∧pro∧cau∧con∧jew to conclusion ¬rob∧¬fou∧dau∧jew (only Case 3 implies the premises), hence also to dau.

6 Concluding remarks

In this paper, we have discussed correct reasoning with evidence using three analytic tools: arguments, scenarios and probabilities. We proposed a formalism in which the presumptive validity of arguments is defined in terms of case models, and studied some properties (Section 3). We discussed key concepts in the argumentative, scenario and probabilistic analysis of reasoning with evidence in terms of the formalism (Section 4). An example of the gradual development of evidential reasoning was provided in Section 5.

This work builds on a growing literature aiming to formally connect the three analytic tools of arguments, scenarios and probabilities. In a discussion of the anchored narratives theory by Crombag, Wagenaar and Van Koppen [37], it was shown how argumentative notions were relevant in their scenario analyses [28]. Bex [3, 5] has provided a hybrid model connecting arguments and scenarios, and has worked on the further integration of the two tools [4, 6]. Connections between arguments and probabilities have been studied by Hepler, Dawid and Leucari [10] combining object-oriented modeling and Bayesian networks. Fenton, Neil and Lagnado continued this work by developing representational idioms for the modeling of evidential reasoning in Bayesian networks [9]. Inspired by this research, Vlek developed scenario idioms for the design of evidential Bayesian networks containing scenarios [35], and Timmer showed how argumentative information can be extracted from a Bayesian network [25]. Keppens and Schafer [12] studied the knowledge-based generation of hypothetical scenarios for reasoning with evidence, later developed further in a decision support system [22].

The present paper continues from an integrated perspective on arguments, scenarios and probabilities [32]. In the present paper, that integrated perspective is formally developed (building on ideas in [31]) using case models and discussing key concepts used in argumentative, scenario and probabilistic analyses. Interestingly, our case models and their preferences are qualitative in nature, while the preferences correspond exactly to those that can be numerically and probabilistically realized. As such, the present formal tools combine a non-numeric and numeric perspective (cf. [32]'s 'To Catch A Thief With and Without Numbers'). Also the present work does not require modeling evidential reasoning in terms of full probability functions, as is the case in Bayesian network approaches. In this way, the well-known problem of needing to specify more numbers than are reasonably available is addressed. Also whereas the causal interpretation of Bayesian networks is risky [7], our case models come with formal definitions of arguments and their presumptive validity.

By the present and related studies, we see a gradual clarification of how arguments, scenarios and probabiities all have their specific useful place in the analysis of evidential reasoning. In this way, it seems ever less natural to choose between the three kinds of tools, and ever more so to use each of them when practically applicable.

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