16

On Coherent Arguments and Their Inferential Roles

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In this paper, we address coherent arguments and their inferential roles, in particular, the explanatory, predictive, and decisive roles. We take a perspective on the coherence of arguments grounded in cases. Our cases are a kind of coherent clusters of information, as they are encountered in the cognitive sciences (scripts, frames, cases, scenarios). We explain how cases can provide a semantics for three kinds of argument validity: coherence, presumptive validity and conclusiveness, and show how these can be used to distinguish three versions of the inferential roles explanation, prediction and decision. The findings are connected to the triplet of inference types deduction, induction and abduction.

1. INTRODUCTION

In this paper, we address coherent arguments and their inferential roles. We take a perspective on argument coherence building on the idea in the cognitive sciences that a part of our cognitive abilities requires coherent clusters of information.

Such coherent information clusters go by different names—among them scripts, frames, cases, scenarios. An example is the restaurant script (Schank & Abelson, 1977), that provides a cluster of information related to a visit to a restaurant: in a restaurant, there are tables, other guests, a waiter, there will be food and drinks, and finally a bill.

Coherent information clusters can be used in different ways—in particular, they can play a role in explanation, prediction, and decision. The restaurant script is *explanatory* as it can be used to make sense of what is happening in a restaurant (answering a question such as 'Why is that person waving his hand to that other person?'); *predictive* as it can be used to infer expectations of what will happen in a restaurant (answering 'What happens next now we are seated at this table?'); and

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decisive in the sense that it can be used to determine whether a certain place actually is a restaurant (answering the question 'Is this a restaurant or an informal dinner?'). In each of these roles, inferences based on them are defeasible, in the sense that they can have exceptions (Toulmin, 1958; Pollock, 1995). For instance, there can be another explanation for the handwaving than asking for the bill in a restaurant; the waiter may not come after being seated; and a scene that has all requirements of a restaurant may in fact be a scene in a theatre play.

We use coherent clusters of information as the background of a notion of coherent arguments. In the formal model associated with the perspective (of which the main definitions are given in an appendix to this paper), the coherent clusters of information are referred to as cases. In combination with an ordering relation, sets of cases form *case models*, that provide a formal semantics for coherent arguments. More specifically, in the presented notion of argument coherence, the evaluation of an argument as coherent depends on the set of cases in a case model. An argument from premises to conclusions is said to be coherent when the combination of premises and conclusions can occur together in one of the given cases. For instance, an argument from the premise 'We have been seated' to the conclusion 'We expect that the waiter will come soon' is coherent given the case of a restaurant script.

As kinds of evaluation of coherent arguments, we discuss two further notions of argument validity: conclusiveness and presumptive validity. A coherent argument from premises to conclusions is conclusive when its conclusions hold in all cases in which the argument's premises hold. A coherent argument from premises to conclusions is presumptively valid when there is a case implying the case made by the argument that is maximal in the ordering relation of the case model. See the appendix for formal definitions and pointers to relevant sources.

The case model approach to coherent arguments used in this paper has been developed in connection to work on structured defeasible argumentation (Pollock, 1987; Simari & Loui, 1992; Vreeswijk, 1997). Today structured defeasible argumentation is often studied using abstract argumentation (Dung, 1995) as point of departure. Abstract argumentation was originally developed in close connection with nonmonotonic logic (e.g., Makinson, 1994) and logic programming (Bondarenko, Dung, Kowalski, & Toni, 1997). Some proposals (notably Prakken, 2010) expand abstract argumentation by treating the nodes of abstract argumentation as abstractions of arguments with a stepwise support structure (much like the derivations in logic). Other approaches expand abstract argumentation by using a richer language that allows for sentences expressing supporting and attacking reasons

(Verheij, 2005). Yet others are distinct from abstract argumentation by a central place for classical deduction (Besnard & Hunter, 2008). For background on this literature, the reader is referred to Chapter 11 of (van Eemeren, Garssen, Krabbe, Snoeck Henkemans, Verheij, & Wagemans, 2014a). Like that chapter, the present paper focuses more on central ideas and less on formal detail.

The case model approach builds on an ongoing discussion on the rational handling of evidence in courts (Anderson, Schum, & Twining, 2005; Kaptein, Prakken, & Verheij, 2009; Dawid, Twining, & Vasiliki, 2011), where analytic styles using arguments, scenarios and probabilities have been used separately and in combinations (for references and recent work, see, e.g., Verheij, Bex, Timmer, Vlek, Meyer, Renooij, & Prakken, 2016). Of these three analytic styles, scenarios play the role of coherent clusters of information. In this setting, the explanatory role becomes apparent in a scenario that helps to make sense of a body of evidence; the predictive role can be recognized in the use of scenarios as a source of possible avenues of investigation; and the decisive role is seen when a scenario is decided to be sufficiently proven on the basis of the evidence. See also (Verheij, 2017b), where the case model approach is used in the setting of evidential reasoning in order to connect arguments, scenarios and probabilities. In the setting of evidential reasoning, the different inferential roles can be understood as follows. In the explanatory role, the evidence leads to a scenario as an explanation of the evidence; in the predictive role, a hypothetical scenario leads to a prediction about what might follow given the scenario; and in the decisive role, inferences are constrained to the possible scenarios.

In the following, we introduce the perspective on coherent arguments in terms of case models, also discussing the other two kinds of argument validity: conclusiveness and presumptive validity (Section 2). Then we discuss the three inferential roles explanation, prediction and decision in terms of coherent arguments (Section 3). In that section, we use the three kinds of validity to distinguish three kinds of explanation, prediction and decision. In Section 4, we connect the previous findings to the notions of deduction, induction and abduction, in order to illustrate the perspective on coherent arguments in terms of case models, and perhaps also to shed light on how these notions have been addressed in the literature.

2. COHERENT ARGUMENTS

Our notion of coherent arguments starts with arguments of an elementary structure, namely arguments that consist of premises and conclusions without considering further argumentative structure. An example is an argument from the premise that a witness testified she saw the suspect at the crime scene to the conclusion that the suspect indeed was at the crime scene.

The coherence of an argument is connected to what we define as the case made by the argument. For an argument consisting of premises and conclusions, the case made by the argument is the combination of the premises and conclusions of the argument; formally as a logical conjunction. For the example argument, the case made by it is as follows:

> The witness testified she saw the suspect at the crime scene; and the suspect was at the crime scene.

Now we come to our concept of coherent arguments. It corresponds to the coherence of the case made by the argument. We have that an argument is coherent if and only if the case made by the argument is coherent.

The coherence of the case made by an argument corresponds to a theory of which clusters of information are coherent. Formally, we have defined this concept in terms of case models. A case model specifies a set of cases. Each case in the case model expresses a coherent cluster of information. Formally, cases are expressed by a set of logically consistent sentences, that are logically different and pairwise incompatible. The cases in a case model can be thought of as the maximally specific clusters of information that are coherent. More generally, a cluster of information—expressed by a logical sentence—is coherent when there is a case in the case model that logically implies the sentence.

For instance, a case model can contain two cases, as follows:

> Case 1: The witness testified she saw the suspect at the crime scene; and the suspect was at the crime scene.
>
> Case 2: The witness testified she saw the suspect at the crime scene; and the suspect was not at the crime scene.

The first of these cases corresponds to the case made by the example argument we saw above. The case expresses the situation that the witness testimony is in fact true. The second corresponds to the situation that the testimony happens to be false. Both are coherent possibilities, since what is testified by a witness may or may not be true. Note that the two cases as a pair are incompatible. Formally, their conjunction is logically inconsistent, since the suspect can logically not be both at the crime scene and also not at the crime scene.

For determining argument coherence, we only need the cases of a case model. When we discuss different inferential roles, we will also use the other, second element of case models, namely an ordering relation on the set of cases. Formally, this ordering is a total preorder of the set of cases, i.e., a total and transitive relation. Total preorders have the special property that they are indifferent about a choice between qualitative and quantitative methods, in the precise sense that they are exactly the ordering relations that can be represented by a numeric ordering.

For the example case model, there are three different possible choices of total preorder on the two cases, each expressing a different ordering of the cases:

Case 1 $>$ Case 2: The first case is ordered higher than the second. This ordering can be used to represent that it is more probable or more believable that the witness testimony is true than not.

Case 1 $\sim$ Case 2: The first case is ordered equally high as the second. This ordering can be used to represent that it is equally probable or believable that the witness testimony is true or not.

Case 1 $<$ Case 2: The first case is ordered lower than the second. This ordering can be used to represent that it is less probable or less believable that the witness testimony is true than not.

Given a case model consisting of cases and their ordering, we can define the notion of presumptively valid arguments. An argument—as before consisting of premises and conclusions—is presumptively valid if and only if two conditions are both fulfilled:

i. The argument is coherent, i.e., there is a case C that logically implies the case made by the argument;
ii. The case C is ordered at least as high as all cases that logically imply the argument's premises.

For instance, for the first of the three possible orderings of the two cases in the example—where Case 1 is ordered higher than Case 2—the argument from the witness testimony to the suspect being at the crime scene is presumptively valid, and the argument from the witness testimony to the suspect not being at the crime scene is not presumptively valid. Both these arguments are coherent though.

Case models can also be used to define the conclusiveness of arguments. An argument is conclusive if and only if two conditions are both fulfilled:

iii. The argument is coherent;
iv. All cases that logically imply the argument's premises imply the argument's conclusions.

As for the coherence of arguments, the conclusiveness of arguments is independent of the ordering relation on the cases. Of the three kinds of argument validity that we have discussed—coherence, presumptive validity and conclusiveness—, only presumptive validity depends on the ordering relation on cases.

3. INFERENTIAL ROLES

In this section, we discuss how the coherent arguments approach can be used to model inferential roles. In the introduction, we mentioned explanatory, predictive, and decisive inferential roles. For each, we give a brief characterization and an example. We show how the examples have the same formal structure and apply the three kinds of argument validity to the inferential roles.

v. In the *explanatory inferential role* of arguments, an argument goes from premises to an explanation of the premises. The explanatory role is connected to the task of explaining a situation. For instance, the occurrence of smoke coming from a house can be explained by the fireplace having been lit or by a fire.

vi. In the *predictive inferential role* of arguments, an argument goes from premises to a prediction given the premises. The predictive role is connected to the task of predicting what comes next. For instance, given the weather forecast that it will be a pleasant summer day, we predict that in fact it will be a pleasant summer day, although perhaps it turns out to be an unpleasant summer day, e.g., unpleasantly hot and humid.

vii. In the *decisive inferential role* of arguments, an argument goes from premises to a decision given the premises. For instance, when a suspect appears in front of a criminal court, and there is no proof of guilt, it will be decided that he is innocent on the basis of the

> presumption of innocence. When proof sufficient for guilt is provided, the decision will be that he is guilty.

These inferential roles and the examples given can be addressed in terms of the notion of coherent arguments discussed.

THE EXPLANATION EXAMPLE

Case 1: There is no smoke coming from the house.

Case 2: There is smoke coming from the house. The fire place has been lit. There is no fire.

Case 3: There is smoke coming from the house. The fire place has not been lit. There is a fire.

Case 1 > Case 2 > Case 3

The first case represents the situation that there is no smoke, and the other two the situation that there is smoke, with the two different explanations. The ordering of the cases suggests that smoke situations are not so common, and that fires are even less so.

The smoke coming from the house can be explained in two ways: by the lit fire place, or by a fire. The fireplace explanation is preferred to the fire explanation.

THE PREDICTION EXAMPLE

Case 1: The weather forecast does not predict that it will be a pleasant summer day.

Case 2: The weather forecast predicts that it will be a pleasant summer day. It is a pleasant summer day. It is not very hot and humid.

Case 3: The weather forecast predicts that it will be a pleasant summer day. It is not a pleasant summer day. It is very hot and humid.

Case 1 > Case 2 > Case 3

Here the first case represents that the forecast does not predict a nice summer day (more common than the opposite according to the ordering), and the other two the nice summer day prediction with its two different outcomes. The ordering suggests that the prediction being correct is what is expected.

The forecast of a pleasant day can lead two outcomes: it comes true or it doesn't. The prediction is that it comes out true.

THE DECISION EXAMPLE

Case 1: There is no proof of guilt.

Case 2: There is proof of guilt. The proof is insufficient for guilt. The decision is not for guilt, but for innocence.

Case 3: There is proof of guilt. The proof is sufficient for guilt. The decision is for guilt, not for innocence.

Case 1 > Case 2 > Case 3

The first case represents that there is no proof of guilt, which is the normal situation (according to the case ordering). The other two cases are the ones where there is proof of guilt, either sufficient or not.

The examples have been designed in such a way that they have the same formal structure, as follows. ¬ denotes negation ('not') and ∧ conjunction ('and'). The legend that connects the elementary sentences *p*, *q* and *r* to the three examples is given in Table 1.

FORMAL STRUCTURE OF THE THREE EXAMPLES

Case 1: $\neg p$.

Case 2: $p \wedge q \wedge \neg r$.

Case 3: $p \wedge \neg q \wedge r$.

	p	*q*	*r*
Explanation	There is smoke coming from the house.	The fire place has been lit.	There is a fire
Prediction	The weather forecast predicts that it will be a pleasant summer day.	It is a pleasant summer day.	It is very hot and humid.
Decision	There is only some evidence for guilt.	The decision is for innocence, not for guilt.	The evidence is sufficient for guilt

Table 1 – Legend for the formal structure of the three examples

Case 1 > Case 2 > Case 3

The three kinds of argument validity—coherence, presumptive validity and conclusiveness—can each be applied to the three inferential roles, given three variants of explanatory, predictive and decisive inference.

viii. EXPLANATION. In *coherent explanation*, inference goes from premises to an explanation that follows coherently, i.e., to any one of all possible explanations. In the example, both the fire place and the fire are coherent explanations of the smoke.
In *presumptive explanation*, inference goes from premises to an explanation that follows presumptively, i.e., an explanation that is maximal in the ordering. In the example, the fire place is a presumptive explanation of the smoke.
In *conclusive explanation*, inference goes from premises to an explanation that follows conclusively, i.e., to an explanation that has no alternatives. In the example, the smoke has no conclusive explanation. But when there is smoke and there is no fire, the lit fire place is the conclusive explanation.

ix. PREDICTION. In *coherent prediction*, inference goes from premises to a prediction that follows coherently, i.e., to any one of all possible ones. In the example, both the pleasant summer day and the unpleasant, very hot and humid day are coherent predictions given the forecast.

In *presumptive prediction*, inference goes from premises to a prediction that follows presumptively, i.e., a prediction that is maximal in the ordering. In the example, the pleasant summer day is a presumptive prediction given the forecast.

In *conclusive prediction*, inference goes from premises to a prediction that follows conclusively, i.e., to a prediction that has no alternatives. In the example, the forecast does not give rise to a conclusive prediction. But given the forecast and the fact that it is not very hot and humid, it can be conclusively predicted that it is a pleasant summer day.

x. DECISION. In *coherent decision*, inference goes from premises to a decision that follows coherently, i.e., to any one of all possible decisions. In the example, both innocence and sufficiency for guilt are coherent decisions given some evidence for guilt.

In *presumptive decision*, inference goes from premises to a decision that follows presumptively, i.e., a decision that is maximal in the ordering. In the example, there is only some evidence for guilt, which leads to the presumptive decision of innocence (since it is possible that the proof is not sufficient for guilt).

In *conclusive decision*, inference goes from premises to a decision that follows conclusively, i.e., to a decision that has no alternatives. In the example, there is only some evidence for guilt, which gives no conclusive decision. But when there is some evidence for guilt and the evidence is not sufficient, the decision for innocence is conclusive.

The reader will have noted that—as a consequence of the equal formal structure of the examples used—the texts follow the same abstract structure, for explanation, prediction and decision. In fact, formally, the remarks about coherent, presumptive and conclusive explanation, prediction and decision can be abstractly phrased as follows. We have only allowed minor variations in the informal descriptions above.

- The two arguments from *p* to *q* and to *r* are both coherent.

- The argument from p to q is presumptively valid, the argument from p to r is not. Also the arguments from p to $\neg r$ and to $q \wedge \neg r$ are presumptively valid.

- The arguments from p to q and to $\neg r$ are not conclusive, the argument from $p \wedge \neg r$ to q is.

4. DEDUCTION, ABDUCTION, INDUCTION

In this section, we put our treatment of inferential roles using coherent arguments grounded in case models in perspective by discussing the well-known triplet of kinds of logical inference: deduction, abduction, induction. Terminology is notoriously non-standard in this connection, with inconsistent positions sometimes fiercely defended in different communities, so we ask the reader for some lenience when reading the following.

Deduction is by many distinguished from induction and abduction by deductive inference being necessary, while induction and abduction are non-necessary. Put otherwise, in deduction, the premises guarantee the conclusion, whereas in induction and abduction they don't. A classic example of deduction is the inference "All men are mortal. Socrates is a man. So, Socrates is mortal". In this example, the generalization is universal, but the term deduction has also been used in connection with the application of non-universal generalizations, as in 'Students speak English. Mary is a student. So, Mary speaks English'. It is then accepted that deductive inference can be defeasible, as, in the example, where Mary may be an exception to the rule that students speak English.

Induction is typically thought of as being based on data. For many, induction is inference to a generalization, as in 'These students all speak English. So all students speak English', or, when granting the existence of non-universal generalizations, '90% of these students speak English. So students generally speak English'. This also shows that induction can have numeric elements, in particular statistical, but can also be phrased entirely qualitatively. Sometimes induction is connected to the application of a pattern grounded in data to a new example, as in '90% of students in past university classes speak English. Mary is a student in the current university class. So, Mary speaks English'. This kind of inference is closely related to the defeasible kind of deduction based on the application of a non-universal rule.

Abduction refers to inference to an explanation. Abduction can be limited to generating any explanation among a number of possibilities, or can be thought of as also selecting a particular choice of explanation. In that connection, one speaks of abduction as inference to the best explanation. For instance, 'Mary speaks English. So, perhaps she is a native speaker of English'.

How do these remarks about the triplet deduction, abduction, induction relate to our discussion of the inferential roles explanation, prediction and decision?

Our inferential role of explanation is directly connected to abduction. We discussed three kinds of explanation—coherent, presumptive and conclusive. Coherent explanation is connected to the idea of abduction as inference to any explanation, and presumptive explanation to inference to the best explanation. Here it should be borne in mind that our presumptive explanation allows for more than one best explanation, namely when there are different cases that follow presumptively, all equivalent in the ordering relation. Our notion of conclusive explanation refers to the kind of abduction where there is exactly one explanation left. Whereas variants of inference to any explanation/coherent explanation and inference to the best explanation/presumptive explanation are extensively discussed in the literature (e.g. Douven, 2017), inference to the only explanation left seems to have received less attention (but see Dawid, Hartmann, & Sprenger, 2015). Interestingly, this is the kind of inference that Sherlock Holmes refers to as deduction, when he says 'When you have eliminated the impossible, whatever remains, however improbable, must be the truth'. It should be noted that our conclusive explanation has the property that it is defeasible since a unique explanation can become excluded by further information. Formally, if we look at the example discussed in Section 3, we have that $\neg r$ follows conclusively from $p \wedge q$, but it does not follow conclusively—not even coherently—from $p \wedge q \wedge r$. This can happen since conclusive arguments have incoherent premises as defeating circumstances. Here premises are considered to be incoherent when the argument from the premises to themselves is not coherent.

The inferential role of prediction is connected to the kind of induction that uses a generalization that is grounded in data. A case model can be thought of as providing the data. Arguments consisting of pairs of premises and conclusions can then be considered as the generalizations grounded in such data. The three kinds of validity can be regarded as different strengths of the generalizations, where coherence is weaker than presumption, which in turn is weaker than conclusiveness. We mentioned that induction sometimes uses numbers, but also is treated in qualitative terms. This connects to the fact that the case model approach has equivalent qualitative and quantitative characterizations, and was in fact inspired by the puzzle of connecting qualitative and quantitative reasoning styles. In this connection, the induction of a coherent generalization can be thought of as corresponding to a positive conditional probability in the data, and the induction of a conclusive generalization to a conditional probability of 100%. The induction of a presumptive generalization corresponds to a maximal conditional probability. As an example, we can consider the case model discussed in Section 3. In one quantitative realization of the case model—there are many—, we have the following:

Case 1: $\neg p$ (90%).

Case 2: $p \wedge q \wedge \neg r$ (9%).

Case 3: $p \wedge \neg q \wedge r$ (1%).

This is a quantitative realization of the case model in Section 3, since the percentages connected to the cases correspond to the ordering Case 1 > Case 2 > Case 3. We now have the following:

- The two arguments from p to q and to r are both coherent. The first corresponds to a conditional probability of 9%/10% = 90%, the latter to 1%/10% = 10%.

- The argument from p to q is presumptively valid, since 90% is larger than 10%. The argument from p to r is not presumptively valid, for the same reason.

- The argument from p to q is not conclusive since 90% is smaller than 100%. The argument from $p \wedge \neg r$ to q is conclusive, since it corresponds to a conditional probability of 9%/9% = 100%.

We come to the third inferential role that we distinguished: decision. The inferential role of decision seems to intuitively be most closely connected to deduction, in particular when we think of deduction as guaranteeing its conclusion. However, we saw that our setting decision has three variations: coherent decision, presumptive decision and conclusive decision. Only conclusive decision is connected to the idea of deduction as guaranteeing a conclusion. In this connection, we already saw that deduction is sometimes thought of as a defeasible form of inference, in particular when the generalization used allows for exceptions. Such defeasible versions of deduction are connected to coherent and presumptive decision, which have exceptions when there are alternative coherent or presumptive possibilities. Note however that also conclusive decision is defeasible, but now not because there is an alternative possibility, but because the only possibility becomes excluded.

The triplet deduction, abduction, induction can also be discussed in connection with variations of classical syllogistic reasoning of the kind 'All men are mortal. Socrates is a man. So, Socrates is mortal'. Each of the triplet deduction, abduction, induction can then be connected to a different ordering of the same three elements, as follows:

- Students speak English.
  Mary is a student.
  So (deductively), Mary speaks English.

- Students speak English.
  Mary speaks English.
  So (abductively), Mary is a student.

- Mary is a student.
  Mary speaks English.
  So (inductively), students speak English.

The first is deductive in the sense that it is rule-following. If the rule (the generalization) is universal, the inference is also deductive in the sense of guaranteeing its conclusion.

The second is abductive in the sense that 'Mary is a student' is an explanation of 'Mary speaks English' in light of the rule 'Students speak English'. The backward application of a rule points to an explanation. This kind of inference is connected to reasoning that is fallacious in the sense of classical logical validity (cf. the fallacy of affirming the consequent). As there can be many explanations for a phenomenon, such abductive inference is generally defeasible.

The third can be regarded as inductive in the sense that the example of the English-speaking student Mary is used to infer a generalization. Since here the inductive inference is based on a single example, the inferred generalization does not have a strong empirical backing. The example can be thought of as being used as a kind of prototype or examplar with typical properties, much like what we see in case-based reasoning. It is also noteworthy that, since the single example does not suggest a direction (there is no distinction between 'Mary is a student' and 'Mary speaks English' that suggests directionality), the rule 'English speakers are students' could be inductively inferred as well.

Can these three examples be interpreted in the setting of case models? In the example, the rule-following character of deduction is emphasized. Each of the three kinds of argument validity can be interpreted in this way. More precisely, we have the following three forms of inference:

> The argument from p to q is coherent/presumptively valid/conclusive.
> p.
> So (deductively), p.

Here the argument is used as a rule that can be applied. The different forms of argument validity can be thought of as correspond to different levels of strength of the rule. One could say that because the rule is valid—when interpreted as an argument—the rule can be applied.

The abductive inference points to the question how the validity of an argument is related to the validity of the argument in which the premises and conclusions have been switched. The following hold in the case model approach:

xi. When an argument from premises φ to conclusions ψ is coherent, its reverse argument from premises ψ to conclusions φ is also coherent.

xii. When an argument from premises φ to conclusions ψ is presumptively valid, its reverse argument from premises ψ to conclusions φ can be presumptively valid, but maybe is not.

xiii. When an argument from premises φ to conclusions ψ is conclusive, its reverse argument from premises ψ to conclusions φ can be conclusive, but maybe is not.

So of the three kinds of valid arguments, only coherent arguments can be safely reversed, keeping coherence. But now recall that conclusive and presumptively valid arguments are coherent. As a result, we also have the following:

xiv. When an argument from premises φ to conclusions ψ is presumptively valid, its reverse argument from premises ψ to conclusions φ is coherent.

xv. When an argument from premises φ to conclusions ψ is conclusive, its reverse argument from premises ψ to conclusions φ is coherent.

We find that all three kinds of valid argument can be reversed, at the price of possibly moving to the weakest of the three kinds of validity: coherence.

5. CONCLUSION

In this paper, we have used a recently developed perspective on coherent arguments grounded in cases in order to shed light on three inferential roles: explanation, prediction and decision. By using the three flavours of argument validity in the perspective—coherence, presumptive validity and conclusiveness—, we could distinguish three versions of explanation, prediction and decision. We connected the approach to the triplet deduction, abduction and induction, thereby illustrating the perspective on coherent arguments and inferential roles, and also shedding different light on the sometimes confusing usage of this terminology.

REFERENCES

Anderson, T., Schum, D., & Twining, W. (2005). *Analysis of evidence* (2nd ed.). Cambridge: Cambridge University Press.

Besnard, P., & Hunter, A. (2008). *Elements of argumentation*. Cambridge, MA: MIT Press.

Bondarenko, A., Dung, P. M., Kowalski, R. A., & Toni, F. (1997). An abstract, argumentation-theoretic approach to default reasoning. *Artificial Intelligence*, *93*(1-2), 63–101.

Dawid, A. P., Twining, W., & Vasiliki, M. (Eds.). (2011). *Evidence, inference and enquiry*. Oxford. Oxford University Press.

Dawid, R., Hartmann, S., & Sprenger, J. (2015). The no alternatives argument. *The British Journal for the Philosophy of Science*, *66*(1), 213–234.

Douven, I. (2017). Abduction. In Zalta, E. N. (Ed.), *The Stanford encyclopedia of philosophy*. Retrieved from <https://plato.stanford.edu/archives/sum2017/entries/abduction/>

Dung, P. M. (1995). On the acceptability of arguments and its fundamental role in nonmonotonic reasoning, logic programming and n-person games. *Artificial Intelligence*, *77*(2), 321–357.

Eemeren, F. H. van, Garssen, B., Krabbe, E. C. W., Snoeck Henkemans, A. F., Verheij, B., & Wagemans, J. H. M. (2014a). Argumentation in artificial intelligence. In *Handbook of argumentation theory* (pp. 615-675) Springer, Berlin.

Eemeren, F. H. van, Garssen, B., Krabbe, E. C. W., Snoeck Henkemans, A. F., Verheij, B., & Wagemans, J. H. M. (2014b). *Handbook of Argumentation Theory*. Springer, Berlin.

Kaptein, H., Prakken, H., & Verheij, B. (Eds.). (2009). *Legal evidence and proof: Statistics, stories, logic*. Farnham: Ashgate.

Makinson, D. (1994). General patterns in nonmonotonic reasoning. In Gabbay, D. M., Hogger, C. J., & Robinson, J. A. (Eds.), *Handbook of logic in artificial Intelligence and Logic Programming* (Vol. 3, pp. 35–110). Oxford: Clarendon Press.

Pollock, J. L. (1987). Defeasible reasoning. *Cognitive Science*, *11*(4), 481–518.

Pollock, J. L. (1995). *Cognitive carpentry: A blueprint for how to build a person*. Cambridge, MA: MIT Press.

Prakken, H. (2010). An abstract framework for argumentation with structured arguments. *Argument and Computation*, *1*(2), 93–124.

Roberts, F. S. (1985). *Measurement theory with applications to decisionmaking, utility, and the social sciences*. Cambridge: Cambridge University Press.

Schank, R., & Abelson, R. (1977). *Scripts, plans, goals and understanding: An inquiry into human knowledge structures*. Hillsdale, Lawrence Erlbaum.

Simari, G. R., & Loui, R. P. (1992). A mathematical treatment of defeasible reasoning and its applications. *Artificial Intelligence*, *53*(2-3), 125–157.

Toulmin, S. E. (1958). *The uses of argument*. Cambridge: Cambridge University Press.

Verheij, B. (2005). *Virtual arguments. On the design of argument assistants for lawyers and other arguers*. The Hague, T.M.C. Asser Press.

Verheij, B. (2014). To catch a thief with and without numbers: Arguments, scenarios and probabilities in evidential reasoning. *Law, Probability and Risk*, *13*(3-4), 307–325.

Verheij, B. (2016a). Correct grounded reasoning with presumptive arguments. In Michael, L., & Kakas, A. (Eds.), *15th European Conference on Logics in Artificial Intelligence, JELIA 2016. Larnaca, Cyprus, November 9–11, 2016. Proceedings (LNAI 10021)* (pp. 481–496). Berlin: Springer.

Verheij, B. (2016b). Formalizing value-guided argumentation for ethical systems design. *Artificial Intelligence and Law*, *24*(4), 387–407.

Verheij, B. (2017a). Formalizing arguments, rules and cases. In *Proceedings of the 16th International Conference on Artificial Intelligence and Law (ICAIL 2017)* (pp. 199–208). New York: ACM Press.

Verheij, B. (2017b). Proof with and without probabilities: Correct evidential reasoning with presumptive arguments, coherent hypotheses and degrees of uncertainty. *Artificial Intelligence and Law*, *25*(1), 127–154.

Verheij, B., Bex, F. J., Timmer, S. T., Vlek, C. S., Meyer, J. J., Renooij, S., & Prakken, H. (2016). Arguments, scenarios and probabilities: Connections between three normative frameworks for evidential reasoning. *Law, Probability and Risk*, *15*(1), 35–70.

Vreeswijk, G. A. W. (1997). Abstract argumentation systems. *Artificial Intelligence*, *90*(1-2), 225–279.

iv.

v.

vi. APPENDIX: FORMAL DEFINITIONS

The case model formal model used here was first presented in (Verheij, 2016a), formalizing a semi-formal presentation in (Verheij, 2014). The formalism was inspired by the setting of reasoning with evidence, where qualitative and quantitative reasoning methods are used. The case model formalism was applied to the combination of arguments, scenarios and probabilities as tools in evidential reasoning (Verheij, 2017b), to value-guided argumentation in the context of ethical systems design (Verheij, 2016b) and to the modelling of reasoning with arguments, rules and cases in the law (Verheij, 2017a). Here we repeat core definitions, referring to the other publications for additional explanation and context (including connections to related literature).

The formalism uses a classical logical language L generated from a set of propositional constants in a standard way. We write $\neg$ for negation, $\wedge$ for conjunction, $\vee$ for disjunction, $\leftrightarrow$ for equivalence, $\top$ for a tautology, and $\bot$ for a contradiction. The associated classical, deductive, monotonic consequence relation is denoted $\models$. We assume a language generated by a finite set of propositional constants.

The central definition is that of case models. The cases in a case model must be logically consistent, mutually incompatible and different; and the comparison relation must be total and transitive (hence is what is called a total preorder, commonly modelling preference relations; Roberts, 1985).

Definition 1. *A* case model *is a pair* $(C, \geq)$ *with finite* $C \subseteq L$, *such that the following hold, for all* φ, ψ *and* $\chi \in C$:

1. $\not\models \neg \varphi$;
2. *If* $\not\models \varphi \leftrightarrow \psi$, *then* $\models \neg(\varphi \wedge \psi)$;
3. *If* $\models \varphi \leftrightarrow \psi$, *then* $\varphi = \psi$;
4. $\varphi \geq \psi$ *or* $\psi \geq \varphi$;
5. *If* $\varphi \geq \psi$ *and* $\psi \geq \chi$, *then* $\varphi \geq \chi$.

The strict weak order > standardly associated with a total preorder ≥ is defined as $\varphi > \psi$ if and only if it is not the case that $\psi \geq \varphi$ (for φ and $\psi \in C$). When $\varphi > \psi$, we say that φ is (strictly) preferred to ψ.

The associated equivalence relation $\sim$ is defined as $\varphi \sim \psi$ if and only if $\varphi \geq \psi$ and $\psi \geq \varphi$.

Although the preference relations of case models are qualitative, they correspond precisely to the relations that can be represented by real-valued functions, hence provide a formally optimal balance between a qualitative and quantitative representation. A numeric representing function can be chosen to formally behave like a probability function.

Next we define arguments from premises $\varphi \in L$ to conclusions $\psi \in L$.

> **Definition 2**. *An* argument *is a pair* (φ, ψ) *with* φ *and* $\psi \in L$. The sentence φ *expresses the argument's premises, the sentence* ψ *its conclusions, and the sentence* $\varphi \wedge \psi$ *the case made by the argument. Generalizing, a sentence* $\chi \in L$ *is a premise of the argument when* $\varphi \models \chi$*, a conclusion when* $\psi \models \chi$*, and a* position *in the case made by the argument when* $\varphi \wedge \psi \models \chi$*. An argument* (φ, ψ) *is properly presumptive when* $\varphi \not\models \psi$*; otherwise non-presumptive. An argument* (φ, ψ) *is an presumption when* $\models \varphi$*, i.e., when its premises are logically tautologous.*

Note our use of the plural for an argument's premises, conclusions and positions. This terminological convention allows us to speak of the premises p and $\neg q$ and conclusions r and $\neg s$ of the argument $(p \wedge \neg q, r \wedge \neg s)$. Also the convention fits our non-syntactic definitions, where for instance an argument with premise χ also has logically equivalent sentences such as $\neg\neg\chi$ as a premise.

A coherent argument is defined as an argument that makes a case that is logically implied by a case in the case model.

> **Definition 3.** *(Coherent arguments) Let* $(C, \geq)$ *be a case model.* Then we define, for all φ and $\psi \in L$:
>
> $(C,\geq) \models (\varphi,\psi)$ *if and only if* $\exists \omega \in C\colon \omega \models \varphi \wedge \psi$.
>
> *We then say that the argument from* φ *to* ψ *is coherent with respect to the case model*.

A conclusive argument is a coherent argument, for which all cases in the case model that imply the argument's premises also imply the argument's conclusions.

> **Definition 4.** *(Conclusive arguments) Let* $(C, \geq)$ be a case model. Then we define, for all φ and $\psi \in L$:

$(C, \geq) \models \varphi \Rightarrow \psi$ *if and only if* $\exists \omega \in C$: $\omega \models \varphi \wedge \psi$ and $\forall \omega \in C$: *if* $\omega \models \varphi$, *then* $\omega \models \varphi \wedge \psi$.

We then say that the argument from φ to ψ is conclusive with respect to the case model.

The notion of presumptive validity considered here is based on the idea that some arguments make a better case than other arguments from the same premises. More precisely, an argument is presumptively valid if there is a case in the case model implying the case made by the argument that is at least as preferred as all cases implying the premises.

Definition 5. *(Presumptively valid arguments) Let* $(C, \geq)$ *be a case model. Then we define, for all* φ and $\psi \in L$:

$(C, \geq) \models \varphi \rightsquigarrow \psi$ *if and only if* $\exists \omega \in C$:

$\omega \models \varphi \wedge \psi$; and
$\forall \omega' \in C$: *if* $\omega' \models \varphi$, then $\omega \geq \omega'$.

We then say that the argument from φ to ψ is presumptively valid with respect to the case model. A presumptively valid argument is properly defeasible, when it is not conclusive.

The three notions of validity (coherence, conclusiveness, presumptive validity) are related, as follows. Conclusive arguments are coherent, but there are case models with a coherent, yet inconclusive argument. Conclusive arguments are presumptively valid, but there are case models with a presumptively valid, yet inconclusive argument. Presumptively valid arguments are coherent, but there are case models with a coherent, yet presumptively invalid argument.

For presumptively valid arguments, we define defeating circumstances, as follows. We distinguish three kinds: rebutting, undercutting and excluding circumstances (cf. the distinction of rebutting and undercutting defeaters; Pollock, 1987, 1995).

Definition 6. *(Defeating circumstances) Let* $(C, \geq)$ *be a case model, and* (φ, ψ) *a presumptively valid argument. Then circumstances* χ *are defeating or successfully attacking when* $(\varphi \wedge \chi, \psi)$ *is not presumptively valid. Defeating circumstances are rebutting when* $(\varphi \wedge \chi, \neg\psi)$ *is presumptively valid;*

otherwise they are undercutting. Defeating circumstances are excluding when $(\varphi \wedge \chi, \psi)$ is not coherent.

Commentary on Verheij's On Coherent Arguments and Their Inferential Roles

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1. INTRODUCTION

In "On Coherent Arguments and Their Inferential Roles" Bart Verheij uses his case model approach to formally represent the inferential roles of explanation, prediction, and decision in argumentation. The case model approach is a model-theoretical account of argument validity that draws on insights from legal argumentation. Inspired by discussions on the handling of evidence in courts, Verheij draws a formal distinction between coherent arguments, conclusive arguments, and presumptively valid arguments. In Section 2 of this commentary I situate Verheij's case model approach by comparing it to an alternative model-theoretical perspective on the formalisation of defeasible inference, focussing on the notion of presumptive validity. In Section 3 I zoom in on the formal handling of two of the three inferential roles discussed by Verheij: explanation and prediction.

2. SITUATING CASE MODELS

Arguments on Verheij's account are premise-conclusion pairs of the form (φ,ψ). They are evaluated against the backdrop of a *case model*: a pair $(C,\leq)$ of a finite set of formulas ("cases") C and a total pre-order $\leq$ on C. An argument (φ,ψ) is *presumptively valid* if ψ is true in at least one of the best φ-cases, where 'best' means "maximal with respect to the ordering $\leq$".

This account of the evaluation of presumptive arguments connects to a broader model-theoretical perspective on defeasible reasoning in which inferences or conditionals are evaluated by checking for their validity relative to an ordered set of (sets of) formulas. Arguably, the best known representative of this perspective is the

Oswald, S. & Maillat, D. (Eds.) (2018). *Argumentation and Inference: Proceedings of the 2nd European Conference on Argumentation, Fribourg 2017* (Vol. I, 407-412). London: College Publications.

"KLM" semantics of Kraus, Lehmann & Magidor (1990). On the KLM-semantics, inferences are evaluated by checking whether their conclusion holds true at the best premise-states, just as Verheij evaluates arguments by checking whether their conclusion holds true at the best premise-cases. Still, there are important differences between both approaches.

First, cases in a case model are required to be logically incompatible, while states in a KLM-model need not be. KLM even allow the same state to occur multiple times in the ordering. Second, the demands on the ordering relation ≤ are relaxed on KLM's approach (≤ is not required to be a total pre-order). Third, presumptive validity is defined in the "credulous" sense for case models, while KLM use a "sceptical" approach to defeasible consequence. This difference is best illustrated by means of an example. Suppose our case model contains two best *p*-cases. One of these is a *q*-case, while the other is a ¬*q*-case. On a credulous approach, the argument (*p,q*) is persumptively valid since *q* holds at *some* best *p*-case. On a skeptical approach, the argument (*p,q*) is *not* presumptively valid, since *q* does not hold at *all* best *p*-cases.

These differences between both approaches uncover the specific design choices that set Verheij's formalism apart from other model-theoretical approaches to defeasible reasoning. The requirement that cases be logically incompatible suggests that case models are appropriate for analyzing situations in which every argumentative stance in a debate or discussion conflicts with *all* other stances. The requirement that ≤ is a total pre-order ensures that the ordering is "numerically representable" and that we can assign probabilities to cases (Verheij, 2017). The choice for a credulous rather than a skeptical notion of presumptive validity makes Verheij's account well-suited for legal applications, where it suffices for the defendant in a trial to establish that she is innocent in *some* (but not all) of the best or most likely cases respecting the evidence.

On the one hand, the strong requirements on case models sometimes amount to a loss of generality when compared to related model-theoretical accounts of defeasible reasoning. On the other hand, these requirements make them especially suitable for some specific contexts of application, particularly legal reasoning.

3. ARGUMENTS AND THEIR INFERENTIAL ROLES

In his discussion of the inferential roles that arguments play in case models, Verheij opts for a unifying characterization in which three such roles receive an identical formal representation: explanation, prediction, and decision. An argument (φ,ψ) is

- explanatory if ψ is an explanation of φ,
- predictive if φ is the basis for the prediction that ψ,
- decisive if φ is the basis for the decision that ψ.

In the remainder of this section I will focus exclusively on explanation and (to a lesser extent) prediction.

In his presentation of explanatory arguments, Verheij mentions a traditional view on explanatory inference as having a "backward" logical direction. Following Peirce, we often see abductive inferences characterised in terms of the *backward modus ponens* or *affirming the consequent* scheme (see e.g. Aliseda, 2006):

$$\frac{\begin{array}{c}\psi \\ \text{If } \varphi\text{, then } \psi\end{array}}{\text{Therefore } \varphi}$$

For instance, from "The streets are wet" and "If it rained recently, the streets are wet" we infer that it rained recently. Respecting the direction of the conditional premise in this inference, we can alternatively characterise explanatory arguments as follows: an argument (φ,ψ) is explanatory if φ is an explanation of ψ. Applied to the notion of presumptive validity this leads to characterisation (ii) as opposed to Verheij's (i):

> (i) (φ,ψ) is a presumptively valid explanatory argument if ψ holds in some best φ-case
>
> (ii) (φ,ψ) is a presumptively valid explanatory argument if φ holds in some best ψ-case

In both cases we check whether the explanandum holds in some best explanans-case. The key difference between (i) and (ii) is that in (i) φ is the explanandum and ψ is the explanans, while in (ii) ψ is the explanandum and φ is the explanans.

Verheij's (i) is structurally identical to his characterization of presumptively valid predictive arguments:

> (iii) (φ,ψ) is a presumptively valid predictive argument if ψ holds in all best φ-cases

Verheij's formalization has the benefit of elegance: predictive and explanatory arguments have the same formal structure. But this elegance comes at a price. The "forward" characterization of explanatory inference in (i) does not respect the temporal and causal direction of explanatory inference, while (ii) does: the explanans is both temporally and causally prior to the explanandum. Suppose, for instance, that we are given the conditional "If there are roadworks on the M6, there are huge traffic jams all over London" ($r \rightarrow t$). The corresponding predictive argument is: "There are roadworks on the M6. If there are roadworks on the M6, there are huge traffic jams all over London. Hence there are [or will be] huge traffic jams all over London" ($r, r \rightarrow t / t$). The explanatory argument goes in the opposite direction: "There are huge traffic jams all over London. If there are roadworks on the M6, there are huge traffic jams all over London. So there are roadworks on the M6" ($t, r \rightarrow t / r$). This alternative characerisation in (ii) preserves the *formal* differences in the presumptive validity of predictive and explanatory arguments.

An interesting further question concerns the extent to which we can distinguish, at the level of the formal framework, between *good* and *bad* predictive/ explanatory/ decisive arguments. (This point is independent of the characterization of explanatory inference in terms of (i) or (ii).) Let me illustrate the matter using Verheij's example for explanatory inference. Suppose

- *p* denotes "There is smoke coming from the house",
- *q* denotes "The fireplace has been lit", and
- *r* denotes "There is a fire".

Assume further that our case model features three cases:

Case 1: $\neg p$
Case 2: $p \& q \& \neg r$
Case 3: $p \& \neg q \& r$

The ordering relation is such that Case 1 > Case 2 > Case 3. As Verheij points out, the argument (p,q) is a presumptively valid explanatory argument: *q* holds in Case 2, the unique best *p*-case. But many more arguments are presumptively valid. It is easily checked, for instance, that all of (q,p), $(p,\neg r)$, $(q,\neg r)$, $(\neg q,p)$, $(\neg q,r)$, $(\neg r,p)$, $(\neg r,q)$, and $(p \& q,\neg r)$ are presumptively valid. Hardly any of these constitute good explanatory arguments, so more is needed to filter out good from bad explanatory arguments.

In the literature on abductive inference we often find further constraints on good explanations, both formal (minimality in terms of logical strength, no self-explanation) and informal (sensitivity to background theory, relevance), see e.g. Aliseda (2006). Implementing these constraints in a formal framework is a great technical challenge, and it is an interesting open question to what extent case models can meet this challenge. The same holds true for predictive and decisive inferences. Here too, further constraints will be necessary to filter out good from bad arguments.

4. CONCLUSION

I have discussed Verheij's case model approach to argument validity via a closer look at some of the design choices made in constructing case models, and via the choices made in defining the inferential roles of arguments. I conclude that Verheij's construction of case models is justified in relation to certain specific contexts of application, such as legal reasoning. His representation of the inferential roles of arguments is motivated in part by concerns of elegance. It is an open question – and a path worth exploring – to what extent the case model approach can be enriched so as to represent further constraints on the evaluation of explanatory, predictive, and decisive arguments.

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REFERENCES

Aliseda, A. (2006). *Abductive reasoning: Logical investigations into discovery and explanations* (Vol. 330). Netherlands: Springer.

Kraus, S., Lehmann, D. & Magidor, M. (1990). Nonmonotonic reasoning, preferential models and cumulative logics. *Artificial Intelligence 44*(1-2), 167-207.

Verheij, B. (2017). Proof with and without probabilities. *Artificial Intelligence & Law, 25*(1), 127-154.